78-251

Optical Motion Control Applied to the Atmospheric Cloud Physics Laboratory (ACPL)

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The use of laser radiation pressure for optical motion control (OMC) of spherical water droplets (5-130 μm radius) is discussed. It is shown that a Gaussian beam can be used to manipulate and stabilize individual particles for studying cloud droplet or ice crystal growth and for investigating collision/coalescence phenomena. In the near-zero-gravity environment of the Atmospheric Cloud Physics Laboratory (ACPL) payload on Shuttle/Spacelab, OMC can provide 1) velocities from <0.01 to 5 cm/s; 2) precise positioning (±5 μm) of water droplets up to 130 μm; and 3) simulation of 1-*g* conditions (velocity or acceleration) for droplets of up to ~13 μm. ACPL experiments benefiting from OMC are also discussed.

I. Background and Introduction

Past and Current Work

THE growth of single cloud droplets and the interaction among cloud droplets has long been recognized as an area of investigation that must be pursued in order to more fully understand the formation of precipitation on the microphysical level.[1] The fundamental question is: How do the small drops (1-50 μm) in clouds grow, in the time period of 1 h or less, to the size of raindrops (100-1000 μm)? A NASA-sponsored Shuttle/Spacelab payload, the Atmospheric Cloud Physics Laboratory (ACPL), is currently being designed with the objective of providing some answers to questions of this nature.

Most ACPL objectives beyond the initial nucleation experiments will require the observation of particles greater than a few micrometers for periods of time greater than 100 s. The dynamic requirements of some experiments (e.g., relative velocities between droplets), coupled with the fact that the ACPL will not be in a true zero acceleration environment, dictate that methods will be required for position and motion control of droplets.

A number of motion control methods are available and many of them are being considered for "zero-gravity" applications. Each technique requires specific ambient and particle characteristics; for example: 1) acoustical methods require a gas (nonvacuum) environment and particle sizes approaching the wavelength of the sound (usually above 1 cm), 2) electrostatics and electrodynamics require charged bodies, 3) electromagnetics requires specific conductivity properties, and 4) optical methods (i.e., radiation pressure) require nonabsorbing particle properties. The first of the above methods, operates on all particles within a volume, while the fourth, optical methods, can be preferentially controlled so that individual particles can be uniquely manipulated. The physical properties of water and ice coupled with the requirements of atmospheric microphysics experiments (both particle size and dynamics) indicate that optical motion control (OMC) is a worthy approach.

Until the relatively recent invention of the laser in 1960, radiation pressure was not considered a force to be reckoned with in the laboratory. When the tremendous light intensities available from relatively low-power (~0.1 W) lasers are applied to small particles (~10-μm radius), the resulting forces are competitive with the previously dominant ones, such as radiometric forces (due to thermal gradients in the medium) and Earth's gravity. The applicability of radiation pressure to controlling the movement of small particles is further enhanced by the potential for conducting the experiments in the nearly acceleration-free (~10^{-3} of Earth's gravity) environment of the Spacelab.

The utilization of radiation pressure as a means of levitating small spheres has been successfully demonstrated in the laboratory by Ashkin.[2] His first experiments used an argon laser with an output of a few milliwatts to accelerate micrometer-sized latex spheres which were freely suspended in water. Ashkin[3] later demonstrated the stable levitation of transparent glass spheres (~20-μm diameter) in air and in vacuum (~1 Torr) using a laser beam of ~250 mW. In other experiments a 50-mW laser was used to accelerate ~5-μm-diameter water droplets in air.

During his laboratory work, Ashkin[2] discovered that the spheres used in his experiments were "simultaneously drawn into the beam axis and accelerated in the direction of light." The attraction toward the center of the beam is a result of the Gaussian energy distribution across the beam diameter and is discussed qualitatively in Sec. II. This effect will be very useful in maintaining the precise position of droplets, since there will always be a force tending to keep the droplet in the beam center (similar to a potential well).

The capabilities of optical radiation pressure as a tool for studying the microphysical processes involved in the atmosphere must be evaluated in terms of the experiments. Typical questions to be answered are: What are the droplet sizes of interest? To what velocities must the droplets be accelerated? What are the associated laser power requirements? Are these power requirements compatible with the constraints of the laboratory (in the case of Spacelab)?

Parameters Important to Optical Levitation or Motion Control

The efficiency with which optical radiation pressure can be applied to the levitation and positioning of particles will be determined by the characteristics of the particles, the particle environment, and the laser. These factors are not mutually exclusive, but for the purposes of the following discussions their relationship to the present objective will be treated separately.

Particle Characteristics

The particle characteristics that obviously affect its amenability to levitation include its shape, mass, and optical properties (i.e., reflection, transmission, and absorption). The cloud droplets of interest here deviate insignificantly

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from perfect spheres. The particle mass will determine the applied force necessary to suspend or accelerate it, while the optical characteristics will determine the efficiency with which the incident optical energy is converted into an applied force. Although all energy absorbed by the particle contributes directly to a momentum transfer to the particle, this absorbed energy also contributes to heating of the particle. Excessive heating of the particle will, in the case of a water droplet, increase its evaporation, and if the heating results in a large temperature gradient in the particle, the resulting photophoretic force may become comparable to the applied radiation force. The importance of these considerations, as they apply to optical levitation of water droplets, will be discussed in Sec. II.

Particle Environment

The particle environment is defined as 1) the medium in which the particle is being levitated and 2) the ambient acceleration field (e.g., gravity) which is present in the laboratory. The temperature, pressure, and viscosity are characteristics of the medium that will determine the thermophoretic forces acting on the particle, as well as the maximum velocity a particle can attain for a given force (Sec. II). The opposing acceleration field will determine the laser power required to perform the actual levitation. Figure 1 illustrates the sources and magnitudes of the perturbing acceleration fields expected for Spacelab.

Laser Characteristics

The laser characteristics, other than power, that must be considered include the efficiency of the laser and the optical mode of the beam. The laser efficiency is normally not the controlling factor for experiments conducted in a terrestrial laboratory since laser input power and physical size are usually not a problem. However, for experiments conducted in the Spacelab, power and size requirements may become important. Since the actual output wavelength has little effect on the required levitation power (Sec. II), an important consideration in choosing a laser (e.g., argon vs helium-neon) should be its efficiency.

Selection of the beam mode (TEM_{01} or TEM_{00}) is dictated largely by the requirements of the experiment. Ashkin[4] has successfully levitated spheres using both the TEM_{01} mode (doughnut-shaped beam) and the TEM_{00} mode (Gaussian beam). His data show that with the TEM_{00} mode, spherical particles can be levitated more stably. Since precise positioning is one of the objectives of optical motion control, the obvious choice for consideration is the TEM_{00} mode laser.

II. Evaluation of Critical Factors for OMC

Forces Relevant to OMC

Radiation Pressure Forces

The force on a body due to radiation pressure is given by van de Hulst[5] as

$$F = (W/c)\,Q_{pr} \tag{1}$$

where W is the power incident on the body (watts), c is the speed of light (cm s^{-1}), and Q_{pr} is the efficiency factor for radiation pressure (dimensionless). The value of Q_{pr} can be calculated from the efficiency factors for absorption Q_{abs} and for scattering Q_{sca} as

$$Q_{pr} = Q_{abs} + Q_{sca}\overline{(1-\cos\theta)} \tag{2}$$

where θ is the angle between the scattered light and the transmitted light. Equation (2) indicates that all absorbed radiation contributes directly to momentum transfer, while only the backward components of scattered radiation ($\cos\theta < 0$) contribute. However, the calculations that follow pertain primarily to water droplets in air, so that the radiation pressure due to the Q_{abs} term is negligible in comparison to the other terms; i.e., water droplets are considered essentially transparent at visible wavelengths.[5] Thus Eq. (2) can be written as

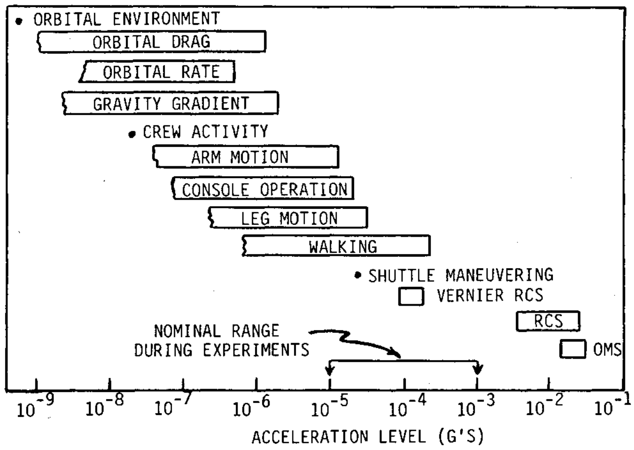

Fig. 1 Shuttle/spacelab acceleration levels.

$$Q_{pr} = Q_{sca}\overline{(1-\cos\theta)} \tag{3}$$

where $Q_{sca} = Q_{ext}$ (extinction efficiency factor) for a nonabsorbing sphere. For the case of water droplets with index of refraction $m = 1.33$ and size parameter $x = 2\pi r/\lambda \gg 1$, van de Hulst[5] gives a value of $Q_{ext} = 2$ (r is drop radius and λ is light wavelength in same units). Equation (3) now becomes

$$Q_{pr} = 2\overline{(1-\cos\theta)} \tag{4}$$

For the above value of m, van de Hulst[5] gives $\overline{(\cos\theta)} = 0.87$, which, when substituted in Eq. (4) gives $Q_{pr} = 0.26$ for spherical water droplets.

The power required to levitate, i.e., exactly counterbalance the gravitational force on a water droplet, is given by setting $F = mg$ in Eq. (1), which then can be written as

$$W = (4/3)\,\pi\rho r^3 gc/Q_{pr} \tag{5}$$

where m has been replaced by $(4/3)\pi\rho r^3$. This gives the levitation power required for a particle radius r and density ρ in the Earth's gravitational field. For a 10-μm radius spherical water droplet, a power of 0.047 W incident on the particle surface is required for levitation.

Dividing the power by the particle cross section gives the required power density (W/cm^2):

$$I = 4\rho rgc/3Q_{pr} \tag{6}$$

Thus, the beam intensity required for levitation is directly proportional to the radius of the droplet. For the example evaluated above ($r = 10^{-3}$ cm), this translates into a power density of approximately 1.5×10^4 W/cm^2, or about 10^5 times the intensity of solar energy on the earth ($\sim$0.14 W/cm^2).

Equations (5) and (6) are general and can be applied to any particle for which the proper value for Q_{pr} is known or can be computed.

Phoretic Forces

A temperature difference over the surface of a body surrounded by a gas will result in a net force on the particle. If the uneven heating of the particle is due to a temperature gradient within the gas, the force is called thermophoretic. If a one-sided illumination of the particle and resulting absorption causes an uneven temperature distribution within the particle, the resulting force is called photophoretic.

Thermophoretic Forces

Consider a sphere residing in a gas in which there is a temperature gradient ∇T_g (e.g., in a thermal diffusion chamber). If the particle radius r is much greater than the mean free paths of the gas molecules, then the temperature gradient within the spherical body is given by Fuchs[6] as

$$\nabla T_p = \frac{3k_g}{2k_g + k_p} (\nabla T_g) \tag{7}$$

where k_g is the gas thermal conductivity (erg cm $^{-1}$ s $^{-1}$ K $^{-1}$), and k_p is the particle thermal conductivity (erg cm $^{-1}$ s $^{-1}$ K $^{-1}$). The thermophoretic force on the particle is then given by Fuchs as

$$F_{\text{th}} = \frac{9\pi k_g}{2k_g + k_p} \frac{\eta^2 r}{\rho_g T_g} (\nabla T_g) \tag{8}$$

where η is the viscosity of gas (g cm $^{-1}$ s $^{-1}$), r is the radius of spherical particle (cm), ρ_g is the density of the gas (g cm $^{-3}$), and T_g is the temperature of the gas (K).

The calculated value for F_{th} below illustrates the effect of the thermophoretic force on a water droplet suspended in air. The following values were assumed for the parameters in Eq. (8):

k_g (air at 294 K) $= 2.56 \times 10^3$

k_p (H_2O at 294 K) $= 5.95 \times 10^4$

η(air at 294 K) $= 1.838 \times 10^{-4}$

ρ_g (air at 294 K) $= 1.201 \times 10^{-3}$

$T_g = 294$ K

The expression for F_{th} then becomes

$$F_{\text{th}} = 1.07 \times 10^{-7} r \nabla T_g \tag{9}$$

Dividing Eq. (9) by the mass of the water droplet, $m = (4/3)\pi\rho r^3$, yields an expression for the thermophoretic acceleration given by

$$a_{\text{th}} = 2.55 \times 10^{-8} (\nabla T_g / r^2) \tag{10}$$

Evaluating the above expression for a 10 μm water droplet radius and a temperature gradient of 1 K/cm yields a thermophoretic acceleration of

$$a_{\text{th}} = 2.55 \times 10^{-2} \text{cm s}^{-2} = 2.6 \times 10^{-5} g$$

where g is the Earth's gravitational acceleration of 980 cm/s $^{-2}$. Equation (10) indicates that the temperature gradients in the medium will have the greatest effect on the smallest particles.

The Stokes velocity associated with this acceleration becomes

$$v_{\text{th}} = \frac{2}{9} \frac{\rho r^2}{\eta} a_{\text{th}} = 3.1 \times 10^{-5} \text{cm s}^{-1} \tag{11}$$

Photophoretic Forces

In the case of a spherical particle illuminated from one side, Fuchs[6] gives the resulting photophoretic force as

$$F_{\text{ph}} = \frac{-3\pi\eta^2 r R_g}{2PM(k_g + k_p)} I\alpha\ell \tag{12}$$

where I is the density of luminous energy flux (erg cm $^{-2}$ s $^{-1}$), R_g is the gas constant (erg K $^{-1}$ mole $^{-1}$), P is the gas pressure (g cm $^{-1}$ s $^{-1}$), M is the molecular weight of the gas (g mole $^{-1}$), α is the absorption coefficient of the particle (cm $^{-1}$), ℓ is the optical absorption thickness (cm $^{-1}$), and all other parameters are as previously defined. A typical value of F_{ph} is calculated by assuming the following values for the parameters in Eq. (12):

$R_g = 8.317 \times 10^7$ erg °C $^{-1}$ mole $^{-1}$
$P = 1.10 \times 10^6$ g cm $^{-1}$ s $^{-2}$
$M = 28.96$ g mole $^{-1}$
$\alpha = 2.9 \times 10^{-3}$ cm $^{-1}$

The computed value for F_{ph} is then

$$F_{\text{ph}} = -2.11 \times 10^{-14} rI\ell \text{ g-cm s}^{-2} \tag{13}$$

Dividing Eq. (13) by the mass of the water droplet gives the acceleration due to the photophoretic force as

$$a_{\text{ph}} = -1.0 \times 10^{-14} (I/r) \tag{14}$$

where a value of $2r$ (worst case) has been assumed for the optical absorption thickness ℓ.

Assuming that the incident levitation power W, as calculated previously, is focused to a beam width equal to the particle diameter, we can write

$$I = W/\pi r^2 \tag{15}$$

and Eq. (14) becomes

$$a_{\text{ph}} = -1.0 \times 10^{-14} (W/\pi r^3) \tag{16}$$

Substituting for W from Eq. (5) gives

$$a_{\text{ph}} = \frac{-1.0 \times 10^{-14}}{\pi r^3} \frac{(4/3)\pi\rho r^3}{Q_{\text{pr}}} gc = -1.54 \times 10^{-3} g \tag{17}$$

which shows that a_{ph} is independent of the particle size. The corresponding Stokes velocity for a 10-μm-radius water droplet in air has a value of

$$v_{\text{ph}} = -1.8 \times 10^{-3} \text{ cm s}^{-1}$$

In the above computations we have assumed conditions relevant to a water droplet suspended in air.

The negative sign associated with the photophoretic force and acceleration indicate that the particle will be propelled in a direction opposite to that of the incident radiation (negative photophoresis), whereas thermophoretic and radiation pressures are in the direction of the beam for water in air. In the case of nearly transparent particles (such as water droplets), this phenomenon results from the fact that the rays refracted inside the particle will cause a greater heating on the side away from the incident light. Highly absorbing particles, on the other hand, will be heated more on the front surface and be moved in the direction of the incident light (positive photophoresis).

Relative Importance of Forces

In order to evaluate the influence of the phoretic forces on the objective of levitating a particle using radiation pressure, the ratios $a_{\text{ph}}/a_{\text{pr}}$ and $a_{\text{th}}/a_{\text{pr}}$ can be calculated. For the case of a 10-μm-radius water droplet being levitated in air by radiation pressure ($a_{\text{pr}} = g$) the values are

$$a_{\text{ph}}/a_{\text{pr}} = -1.5 \times 10^{-3} \text{ and } a_{\text{th}}/a_{\text{pr}} = 2.6 \times 10^{-5}$$

Thus, it can be concluded that for this example of a 10-μm droplet, the phoretic forces can be neglected compared to the radiation pressure force. Furthermore, since a_{th} varies as r^{-2} and a_{ph} is independent of r, they will not be important for larger droplets.

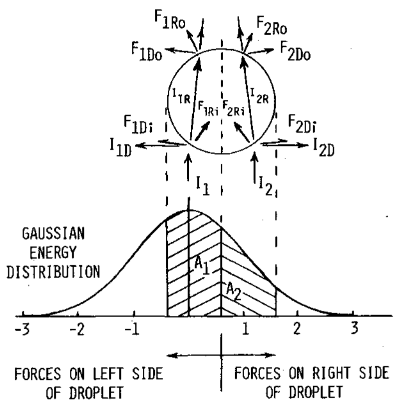

Fig. 2 Illustration of forces acting on a droplet by a Gaussian laser beam.

Transverse Stabilizing Force of the Laser Beam

The forces discussed in the previous sections are important in terms of determining whether or not a particle can be levitated or accelerated using optical radiation. However, given that a laser will suspend a particle, why does the particle remain in the beam and not slowly drift to the side and subsequently get lost? Ashkin[2] observed that particles were actually attracted toward the center of the laser beam (Gaussian mode) and held in position. This phenomenon can be explained qualitatively by referring to Fig. 2.

A droplet being suspended by a laser with a Gaussian energy distribution will experience a variation in the incident energy across its surface. Consider the sketch in Fig. 2, which illustrates the optical paths of two parallel rays, I_l and I_2, of light incident on the droplet. Both rays are equally distant from the droplet center, but I_l originates nearer the center of the beam and its therefore stronger than I_2. The forces resulting on the droplet are determined by drawing the resulting vectors at each droplet-air interface. For example, the incident ray I_l and the reflected ray I_{lD} produce a force F_{lRi}, while the incident ray and the refracted ray I_{lR} produce a force F_{lDi}. Similarly, the forces resulting at the exit point of ray I_l as well as the corresponding forces for ray I_2 can be determined.

As shown in the sketch, all forces contribute to "pushing" the particle in the direction of the incident light. However, forces F_{lDi} and F_{lDo} are directed toward the center of the beam while forces F_{2Di} and F_{2Do} are directed away from the beam center. The forces, F_{lRi} and F_{lRo} cancel to first order. Since the intensity of ray I_l is greater than the intensity of ray I_2, all forces associated with I_l will be greater than the corresponding forces associated with I_2. Thus we can write

$$\left.\begin{matrix} F_{lDi} > F_{2Di} \\ F_{lDo} > F_{2Do} \end{matrix}\right\} \text{ or } F_{lDi} + F_{lDo} > F_{2Di} + F_{2Do} \tag{18}$$

All rays incident on the left half of the droplet (cf Fig. 2) will result in a force directed toward the beam center ($-x$ direction), while rays incident on the right half will result in a force directed out of the beam ($+x$ direction). The relative magnitudes of all rays can be determined by their origin within the Gaussian energy distribution of the beam. For example, I_l originates at the beam center ($x=0$) and will have a magnitude of

$$I_l \propto e^{-x}/\sqrt{2\pi} \simeq 0.4 \tag{19}$$

while I_2 originates at the 2σ point ($x=1$) and will have a value of

$$I_2 \propto e^{-l}/\sqrt{2\pi} \simeq 0.15 \tag{20}$$

However, the total restoring force on the droplet (F_{x-}) will be proportional to the area A_l under the Gaussian energy distribution curve. Similarly, the total force tending to push the droplet out of the beam (F_{x+}) will be proportional to A_2.

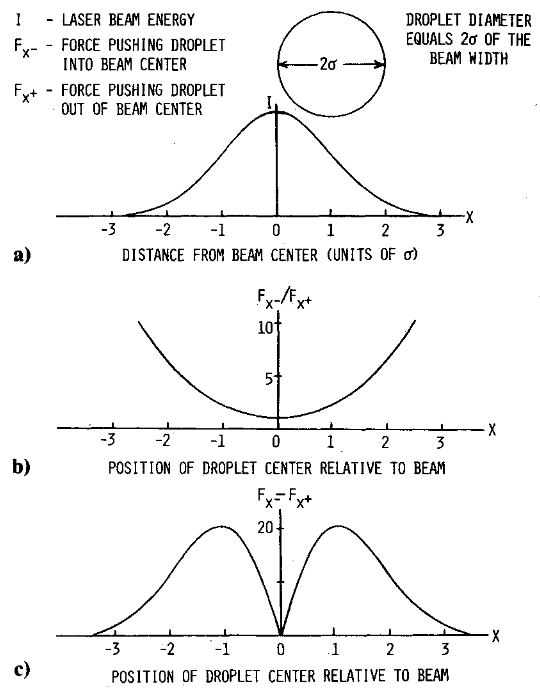

Fig. 3 a) Comparison of 2σ water droplet with Gaussian beam; b) positional variation of restoring force/repelling force (F_{x-}/F_{x+}); c) positional variation of net restoring forces ($F_{x-}-F_{x+}$).

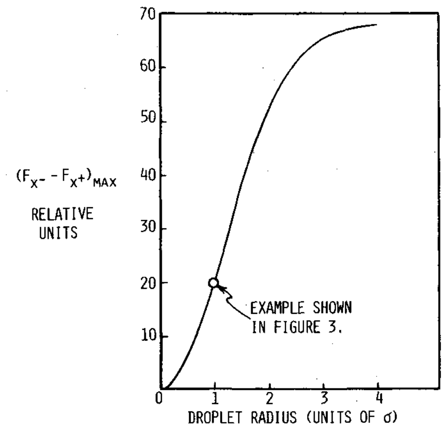

Fig. 4 Variation of maximum restoring force as a function of droplet size. Droplet size is given in terms of laser beam width.

The relative magnitude of these forces can easily be computed for various drop sizes. In Fig. 3a, a droplet of diameter 2σ (relative to the width of the laser beam) is positioned in a Gaussian laser beam. As the droplet is moved across the diameter of the laser beam, the corresponding values of F_{x-}/F_{x+} (i.e., A_l/A_2) are computed in relative units and plotted in Fig. 3b. The ratio has a minimum value of 1 when the droplet is at the beam center and increases as the droplet moves to either side of the center, indicating that F_{x-} is always greater than F_{x+}. Similarly, the net restoring force ($-x$ direction) can be determined qualitatively from a plot of ($F_{x-}-F_{x+}$) as shown in Fig. 3c. The maximum restoring force occurs when the droplet is centered at the 1σ position of the beam corresponding to the point at which the intensity gradient of the Gaussian distribution is a maximum.

As the droplet size increases, the corresponding values of (F_{x-}/F_{x+}) and ($F_{x-}-F_{x+}$) will also increase, indicating that the restoring force on a particle will increase as the droplet size and beam size become comparable. Figure 4 illustrates this effect for $(F_{x-}-F_{x+})_{\max}$, i.e., a droplet positioned at the 1σ point in the beam. It should be noted, however, that the restoring force will become less effective as the droplet becomes so large that it effectively presents a flat surface to the beam.

Heating of Optically Levitated Particles

Consideration must be given to the absorption heating of an optically levitated particle in as much as it may define the limits within which optical levitation is permissible. The eventual steady-state temperature can be computed in a relatively straightforward manner by equating the heat input (absorbed optical energy) and heat loss (radiation and conduction to the medium) for the levitation power required.

The heat equivalent of the incident beam can be expressed by

$$\dot{Q}_{in} = \alpha k(1-q)W \tag{21}$$

where

- $\dot{Q}_{in}$ = heat input to the droplet (cal/s)
- W = total power incident on droplet (erg/s)
- q = fraction of incident beam that is reflected ($\simeq 0.1$)
- $(1-q)$ = fraction of incident beam transmitted ($\simeq 0.9$)
- k = conversion factor from optical energy to mechanical equivalent of heat ($\simeq 0.239 \times 10^{-7}$ cal/erg)
- α = fraction of transmitted beam that is absorbed (2.9×10^{-3} cm^{-1})ℓ
- ℓ = optical absorption thickness (assume equal to drop diameter as a worst case)

Equation (21) can thus be expressed in terms of the levitation power and the droplet size as

$$\dot{Q}_{in} = 1.25 \times 10^{-10} Wr \tag{22}$$

Given sufficient time, the droplet will attain an equilibrium temperature where the heat conduction outward from the center of the spherical drop can be expressed as

$$\dot{Q}_{c_{out}} = -k_g A \, dT/dr = -4\pi r^2 k_g \, dT/dr \tag{23}$$

which will be true for any drop radius (r). In the above expression the parameters are defined as follows: $\dot{Q}_{c_{out}}$ is the rate of outward heat flow (cal/s), r is the radius from center of drop (cm), k_g is the thermal conductivity of media (erg cm^{-1} s^{-1} K^{-1}), T is the absolute temperature (K), and A is the spherical surface area at radius r (cm^2). When Eq. (23) is integrated and evaluated at a large distance from the drop, one obtains the outward heat flow from the spherical surface at temperature T_s, to the conductive medium at temperature T_m. The result is

$$\dot{Q}_{c_{out}} = 4\pi r k_g (T_s - T_m) \tag{24}$$

Using a value of 6.1×10^{-5} cal cm^{-1} s^{-1} K^{-1} for the thermal conductivity of air we obtain

$$\dot{Q}_{c_{out}} = 7.7 \times 10^{-4} r(T_s - T_m) \tag{25}$$

Similarly, the radiative heat loss from the droplet can be calculated in a straightforward manner using

$$\dot{Q}_{r_{out}} = 4\pi\epsilon r^2 \sigma (T_s^4 - T_m^4) \tag{26}$$

where ϵ is the emissivity of the water droplet, and σ is the Stefan-Boltzmann constant (1.36×10^{-12} cal s^{-1} cm^{-2} K^{-4}). The emissivity is given by

$$\epsilon = 1 - e^{-\alpha\ell} \simeq \alpha\ell \text{ for } \alpha\ell \ll 1 \tag{27}$$

where α and ℓ have the values previously given. Substitution into Eq. (26) yields

$$\dot{Q}_{r_{out}} = 9.9 \times 10^{-14} r^3 (T_s^4 - T_m^4) \tag{28}$$

The relative importance of radiative and conductive heat loss from the droplet is readily obtained by dividing Eq. (24) into Eq. (28) to obtain

$$(\dot{Q}_r/\dot{Q}_c)_{out} \simeq 10^{-10} r^2 (T_s^3 + T_s^2 T_m + T_s T_m^2 + T_m^3) \tag{29}$$

At these low temperatures and small particle sizes, the radiative heat loss is insignificant when compared to the conductive heat loss to the surrounding medium (assumed to be air at 1 atm). However, it must be pointed out that if the levitation is attempted at sucessively lower pressures, the conductive effects will decrease until eventually the radiation term becomes dominant.

As a consequence of Eq. (29) the equilibrium temperature of the droplet can be calculated by considering the simplified heat balance equation, $\dot{Q}_{in} = \dot{Q}_{out}$ or

$$1.25 \times 10^{-10} Wr = 7.7 \times 10^{-4} r(T_s - T_m) \tag{30}$$

which is obtained from Eqs. (22) and (25). Solving for the temperature increase of the drop, we have

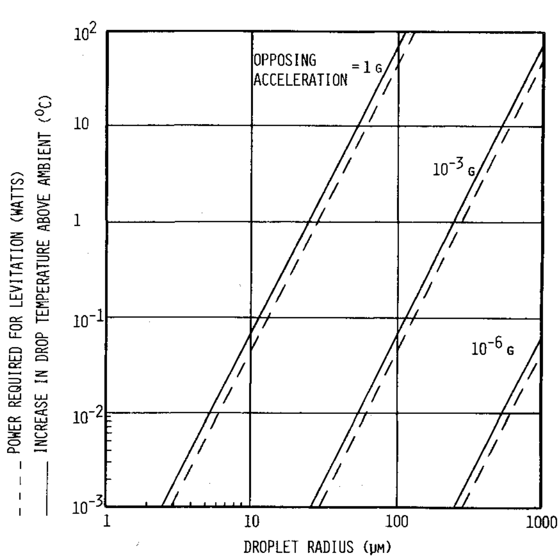

Fig. 5 Required levitation power and corresponding temperature rise for a water droplet suspended in air (T = 294 K, P = 1 atm).

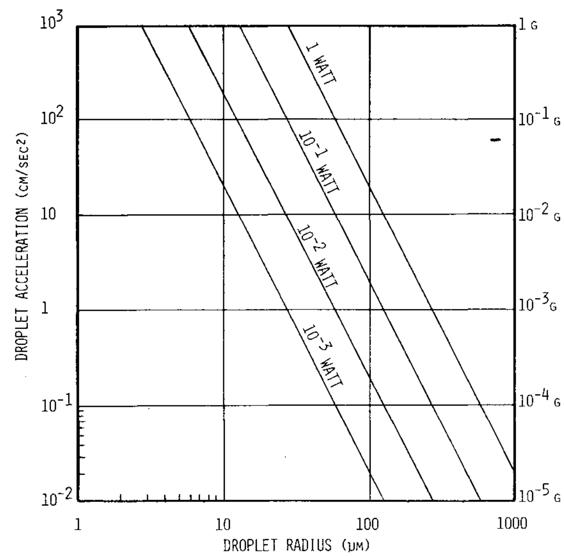

Fig. 6 Acceleration due to radiation pressure as a function of water droplet size for several laser power levels.

$$\Delta T = (T_s - T_m) = 1.6 \times 10^{-7} W \tag{31}$$

where W (the levitation power) was given by Eq. (5). The delta temperature then becomes

$$\Delta T = 7.7 \times 10^4 g r^3 \tag{32}$$

where a value of $\rho = 1$ (water) has been assumed.

Thus, for a 10-μm (radius) spherical water droplet being levitated at standard temperature and pressure conditions in a terrestrial laboratory, a temperature increase of approximately 0.08 °C will be observed. The associated evaporation will result in a net force that is negligible owing to the thermal symmetry of the droplet. A graph of required levitation power and the corresponding temperature rise is given in Fig. 5 as a function of particle size, assuming opposing acceleration fields of 1, 10^{-3}, and 10^{-6} g, where $g = 980$ cm/s^2.

Practical Limits of Acceleration and Velocity

Typical Acceleration Levels

The acceleration that can be imparted to a particle by optical radiation pressure is derived from Eqs. (1) and (5) as

$$a_{pr} = \frac{3}{4c\pi} \frac{Q_{pr}}{\rho r^3} W \tag{33}$$

Substituting the values of $Q_{pr} = 0.26$ and $\rho = 1$ g/cm^3, which are relevant to a water droplet, Eq. (33) becomes

$$a_{pr} = 2.07 \times 10^{-12} W / r^3 \tag{34}$$

The variation a_{pr} with changing values of r and W can now be easily calculated by alternately holding r and W at constant values. Figure 6 illustrates the variation of radiation pressure acceleration (a_{pr}) as a function of droplet radius for several values of laser power input.

In order to obtain the net acceleration of the particle, the opposing acceleration (e.g., earth's gravity) must be subtracted from the values plotted in the figures. For example, Fig. 6 indicates that working in a terrestrial laboratory with a 100-mW laser, an acceleration of ~1 g (980 cm/s^2) can be given to a 14-μm-radius droplet. However, this is just equal to the opposing acceleration of Earth's gravity, so that the net acceleration of the droplet is zero. If the same laser were used in the Spacelab where a typical acceleration level might be ~10^{-3} g (Fig. 1), the net acceleration of the particle would be almost 1 g.

Wavelength Dependence of Levitation Power

The particle acceleration achieved is primarily a function of the Mie size parameter, $x = 2\pi r/\lambda$, which relates the particle radius (r) and the wavelength (λ). Maximum acceleration is achieved with $x \simeq 1$ with smaller local maxima at multiples of this value.[7] In the case of optical levitation ($\lambda \simeq 6328$ Å) under consideration here, $x \gg 1$ for particles greater than 1 μm. As a result, the value of $Q_{pr} = 0.26$ will remain virtually unaffected as λ is varied within the bounds of the visible spectrum, and the levitation power required will remain constant.

Typical Velocity Levels

A water drop being accelerated in air by a constant applied force will eventually attain an equilibrium velocity (v) determined by the resistive drag force, which can be expressed as follows[8]:

$$F_d = (\pi r^2 / 2) C_d \rho_g v^2 \tag{35}$$

where C_d is the drag coefficient and is a function of the Reynolds number Re:

$$Re = 2 v r \rho_g / \eta \tag{36}$$

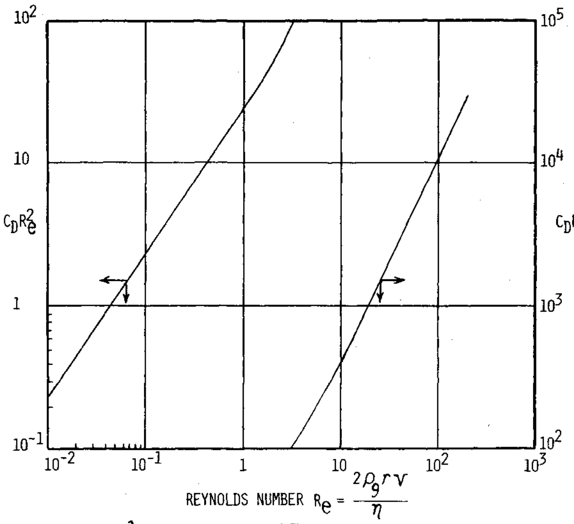

Fig. 7 $C_d Re^2$ as a function of Reynolds number (from Mason[8]).

defined by

$$C_d Re / 24 = F_d / 6\pi\eta r v \tag{37}$$

The drag force can then be expressed as

$$F_d = (C_d Re / 24) 6\pi\eta r v \tag{38}$$

and the force equation for the drop becomes

$$m \frac{dv}{dt} = ma - \left(\frac{C_d Re}{24}\right) 6\pi\eta r v \tag{39}$$

The left-hand side of Eq. (39) is the net force acting on the droplet. The first term on the right is the radiation pressure force [Eq. (1)] acting on the droplet (neglecting the buoyancy of air), while the second term is the drag force. At equilibrium (i.e., $dv/dt = 0$) Eq. (39) can be solved for the velocity to obtain

$$v = (24 / C_d Re)(ma / 6\pi\eta r) \tag{40}$$

Substituting for the particle mass we have

$$v = \frac{24}{C_d Re} \frac{2}{9} \frac{\rho r^2}{\eta} a \tag{41}$$

In the case of Stokes Law ($Re \leq 0.1$), $C_d Re = 24$, and Eq. (37) becomes

$$v = (2\rho/9)(r^2 a / \eta) \tag{42}$$

As Re becomes larger, Stokes Law increasingly over estimates the actual velocity so that empirical relations between $C_d Re$ and Re are usually used. The data of Beard and Pruppacher[9] showed that water drops can be well represented by the following formulas given by Mason[8]:

$$C_d Re / 24 = 1 + 0.10 Re^{0.955}, \quad 0.01 \leq Re \leq 2, \tag{43a}$$

$$= 1 + 0.11 Re^{0.81}, \quad 2 \leq Re \leq 21 \tag{43b}$$

$$= 1 + 0.189 Re^{0.632}, \quad 21 \leq Re \leq 200 \tag{43c}$$

In practice $C_d Re^2$ is usually plotted as a function of Re as shown in Fig. 7. The quantity of $C_d Re^2$ can be expressed in terms of the drop radius, the air (ambient) characteristics, and the acceleration level as[9]

$$C_d Re^2 = \frac{32}{3} r^3 \rho_g \rho \frac{a}{\eta^2} \tag{44}$$

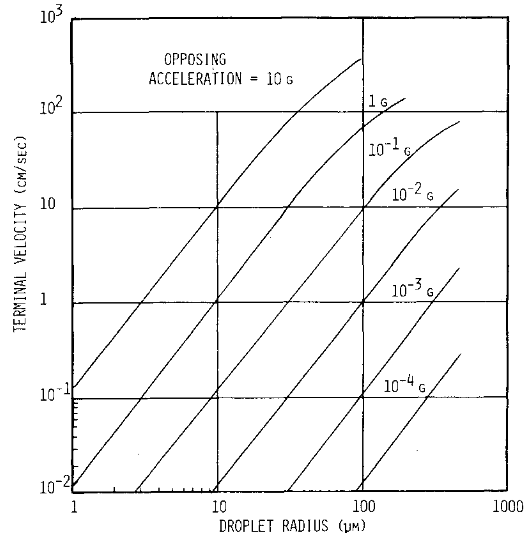

Fig. 8 Water droplet velocity in air ($T = 294$ K, $P = 1$ atm) for several values of opposing acceleration ($g = 980$ cm/s^2).

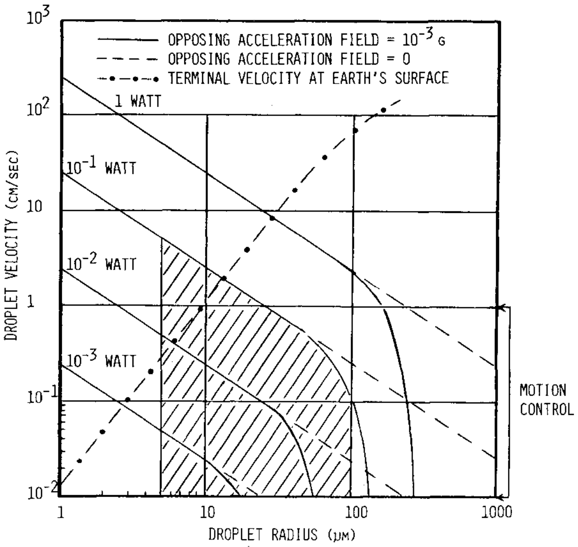

Fig. 9 Water droplet velocity as a function of droplet size. Cross hatched area indicates region of applicability for Shuttle/Spacelab experiments (assuming 100-mW laser).

where a is the net acceleration of the particle. Thus, the procedure for calculating the equilibrium velocity of a water droplet is as follows:

1) Specify the drop characteristics (r,ρ), the ambient characteristics (ρ_g,η), and the net acceleration (a).
2) Calculate $C_d Re^2$ from Eq. (44).
3) Determine the corresponding value of Re from Fig. 7.
4) If $Re \leq 0.1$, calculate v from Eq. (42).
5) If $Re \geq 0.1$, calculate v from Eq. (36).

Figure 8 illustrates the equilibrium or terminal velocity attained by a water droplet in air ($T = 294$ K, $P = 1$ atm) for various multiples of Earth's gravity (g).

In order to utilize laser energy to control the motion and position of a droplet, the acceleration imparted to it must be sufficient to exceed the opposing acceleration field (e.g., Earth's gravity in a terrestrial laboratory). In Fig. 9, the equilibrium velocity has been plotted as a function of droplet radius for various values of laser input power. This figure illustrates the limitation imposed by an opposing acceleration field of 10^{-3} g. The surrounding medium is again assumed to be air at $T = 294$ K and $P = 1$ atm.

The equilibrium velocity that would pertain if there were no opposing acceleration is indicated by the dashed lines. Also shown on the figure is the terminal velocity a droplet would have in the Earth's atmosphere. Thus, if it is desired to simulate droplet collisions resembling those that might be naturally occurring in the atmosphere, the "1 g" dot-dash line shown in the figure can be used as a guide in determining the laser power required for a given drop size. If a particular experiment requires precise positioning of a droplet, the droplet velocity must be relatively low, e.g., <0.1 cm/s.

III. Applicability of OMC to Cloud Physics Studies

ACPL on Shuttle/Spacelab

The previous sections have shown that the utilization of optical motion control (OMC) is primarily determined by the available laser power. The Air Force (among others) is currently developing a space-qualified laser for use in satellite-to-satellite communications which will be compatible for use on Shuttle/Spacelab. This laser will operate at a wavelength of 0.532 μm with a power output of between 100 to 200 mW and will be available by 1980. We can, therefore, assume that the first application of OMC to the ACPL will have a usable power of 100 mW. In addition, we can assume that the maximum residual or ambient acceleration in the Spacelab is 10^{-3} g (cf. Fig. 1). Within these bounds we can define the applicability of OMC for experiments relating to 1) the growth of water droplets or ice crystals and 2) the collision or coalescence of two or more drops or crystals.

The role of OMC in the first of these applications will depend on the method by which the droplets or crystals are generated. For example, they may be generated "in position" within the supersaturated environment of a static diffusion chamber. The laser beam would then be used as a probe to select and hold an individual droplet in position so that its growth history could be observed. An alternate method would be to generate droplets singly (e.g., using a wire probe retractor) near the edge of the chamber and then use the laser energy to move the droplet into a location where it could be fixed in a viewing position by a second laser beam. The droplet size range to which these applications can be applied can be easily visualized by refering to Fig. 9.

Droplet sizes in the range of 5-100-μm radius are of particular interest for cloud physics investigations on the microphysical level. The cross-hatched area in Fig. 9 indicates the maximum velocities that can be expected for an applied laser power of 100 mW. Droplets with radii smaller than ~13 μm can be accelerated to velocities that are greater than their

Table 1 ACPL experiments benefitting from motion control

Experiment	Applicable aspects
Ice multiplication	e.g., collision of ice/ice or ice/water
Charge separation	e.g., collision of ice/ice or ice/water in presence of d.c. electric field
Ice crystal growth habits	to offset the 10^{-3} to 10^{-5} g drift for extended periods and to align crystal for photography
Scavenging	control motion of large ice/water particles through field of smaller particles
Riming and aggregation	dynamic interaction between ice/ice and ice/water requires precise motion control
Collision-induced freezing	motion control to initiate the collision
Supercooled-water saturation vapor pressure	provide the extended time required for equilibrium
Droplet collision	motion control to provide relative velocities
Coalescence efficiencies	motion control to provide known low-acceleration relative motion between particles
Unventilated droplet diffusion coefficient	position control for extended observation periods

respective terminal earth's atmosphere (1-g) velocities. Dynamic processes such as scavenging and collision/coalescence can therefore be simulated and observed in 0 g as they might naturally occur in the 1-g earth environment but with better control and longer observation times. Figure 9 also illustrates that the upper size limit for position control of water droplets will occur at ~130 μm (the size for which the 100-mW curve becomes essentially vertical). This size limit could also be obtained from Fig. 6 by noting the droplet size at which its radiation pressure acceleration is equal to the opposing ambient acceleration of 10^{-3} g.

A summary of potential ACPL experiments which would benefit from the implementation of optical motion control capability is presented in Table 1.

Thus, OMC can provide a significant contribution to atmospheric microphysics experimentation in ACPL through positioning and variable acceleration motion control.

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